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The generalizability of financial distress prediction models: Evidence from Turkey

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Abstract: This study analyzes five of the well-known and most cited distress prediction models in the literature. The models are implemented to continuous publicly listed industrial firms in Turkey through their original and re-estimated coefficients in a comparative way to examine their generalizability in different time periods and samples. The effect of 2008 financial crisis is also assessed to conduct a fuller analysis of the models' prediction accuracies. The results emphasize that Ohlson (1980), Taffler (1983), Zmijewski (1984), and Shumway (2001) provide highly accurate distress classification results through their original coefficients for Turkish industrial market. On the other hand, the re-estimation of the models (other than Ohlson's [1980]) fails to improve the prediction accuracies which are also found insignificant by considering the pre and post crisis periods.

Keywords: Financial distress prediction, emerging markets, model comparison, financial crisis, multiple discriminant analysis, logit, probit, hazard model, financial ratios

JEL codes: M21, C13, C33, C35, C55

1. Introduction

The business capability of a firm is affected by distinct factors. These factors, grouped as managerial, economic, political, and environmental, should be identified to sustain the value creation and survival of firms. Although there are many external factors that are difficult to analyze due to measurement concerns,

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one way to assist firms in planning for future operations is to conduct an elaborate financial analysis based on the financial statements and the financial ratios.

The information provided by the financial statements not only presents the current condition of the firm, but also shed lights on its future (Beaver, 1966), and the ratios calculated from distinct accounts represent a simplified interpretation of the information with a predictive ability (Horrigan, 1968). Therefore, analyzing the financial statements through financial ratios as indicators for the future of the firm would allow the estimation of the risk of failure. On the other hand, considering each financial ratio in isolation may not help understanding the condition of the firm as a whole. Since each ratio accommodates information from distinct sections, these should be conceded simultaneously in order to understand the entire operating ability of the firm. In particular, the studies after 1960s are statistically more sophisticated, enabling the prediction of firm bankruptcy/distress (the term distress will be used throughout of the manuscript) constituting the basis of the current study. The concurrent analysis of the ratios in distress models aims to determine the accuracy of the failure prediction vital due to its high cost for the stakeholders.

There are numerous bankruptcy studies in the accounting and finance literature, applied to both developed and developing markets. In most of these studies, the main concern is to establish a distress classification model, without clearly establishing the need or purpose of the model. The main purpose of these studies is to acknowledge the distress, or even the failure, in advance, so the contribution is on the *"obvious practical interest"* as indicated by Ohlson (1980). In addition to this, the absence of a theoretical base or the so called "*common ground*" for prediction model studies (Grice & Dugan, 2003) does not necessarily imply that these models cannot be generalized in terms of the accuracy levels in which they provide for distinct samples and time periods. Thus, the motivation of the current study is to produce a comparative measure of the accuracy levels of the five well known distress prediction models described in the literature for an emerging economy. The questions that are asked: 1) Is applying these prediction models, derived for developed markets, to an emerging market sample enable the generalizability of these models for different market samples and time periods?; 2) Is there a significant change in terms of the prediction accuracies before and after the financial crisis of 2008?

In order to show evidence of generalizability, the prediction accuracies are measured, and each model is initially applied over its original coefficients and then re-estimated by adhering to its original variables to each of the industrial firms in the sample. In the event that one or more of these models is generalizable, depending on prediction accuracies, it will be conceded that the related model(s) are plausible to use as a benchmark for the establishment of a new prediction model, which would strengthen the so called "*common ground*" for distress prediction studies.

The models of the present study are mainly developed on the US sample by Altman (1968), Ohlson (1980), Zmijewski (1984) and Shumway (2001), and on the UK sample by Taffler (1983) to classify bankrupt and healthy firms. To the authors' knowledge, the current study is the pioneer to conduct a comparative analysis of these five models on a continuing firm sample. The prediction models are implemented to continuous firms, following Altman (1968), for his sub-sample for the period of 1958 to 1961, and Tinoco and Wilson (2013). The continuous firm sample measures the predictive ability of the models on ongoing firms which enables practitioners to understand whether the industrial firms would report negative income in the following consecutive years. Any of the firms reporting negative income for two consecutive fiscal years are considered as distressed (Li and Sun, 2008; Xie *et al*. 2011). Furthermore, analyzing ongoing firms helps researchers overcome some vital concerns mentioned in the literature regarding the incompleteness and the limited number of bankruptcy data (Ohlson, 1980; Zmijewski, 1984).

Few studies have compared the prediction models over their original and the re-estimated versions only for the developed market samples. These were the models of Altman by Grice and Ingram (2001), Altman vs Ohlson by Begley *et al*. (1996), and Zmijewski vs Ohlson by Grice and Dugan (2003). Thus, this study contributes to the literature in three ways: i) The five best known and most cited prediction models used in this study are initially applied over original coefficients and then through their re-estimated versions in order to compare the results in an emerging market sample for their generalizability. ii) Both the original and the re-estimated models are applied to continuous firms in an emerging market sample, rather than those that have failed or are in the bankruptcy process. iii) The original and the re-estimated versions of the prediction models are exercised in a comparative way regarding the effects of the financial crisis of 2008.

2. Literature review

The very first steps in financial distress modeling were taken by Beaver (1966). Beaver's study introduced a univariate analysis of financial ratios to explore whether financial ratios had predictive ability for the financial failure of individual firms; the claim of the study is that the financial ratios are composed of numbers taken from the financial statements that represent both important and non-important events related to the firm. Moreover, Beaver indicated the usefulness of financial ratios in their predictive ability; in other words, the predictive ability of accounting numbers in financial statements were indirectly measured in his study, and were found to be significant. The sample included 79 failed firms and 79 non-failed firms. Non-corporate, privately held and non-industrial firms were not included in the sample. The sample firms were gathered from Moody's Industrial Manual, the only database available. The firms were grouped according to industry and asset size; thus, the method used was to match pair of firms from the sample. Moreover, the sample of the study represented 90% of the invested capital of all industrial firms, and thus had the potential to affect a substantial number of stakeholders. The high level of descriptive detail of the study was important; an in-depth analysis of the six chosen ratios showed that the mean differences of the financial ratios increased as firms approached failure. As a result, Beaver emphasized the importance of financial ratio analysis in warning financially unhealthy firms and preventing becoming bankrupt.

The following study by Altman (1968), one of the most cited studies in the field of financial distress modeling, indicated the importance of financial ratio analysis over bivariate analysis when using the ratios in his model. Bivariate analysis is suitable for measuring the simultaneous contribution of the ratios to the explanatory power of the model, which makes Altman's study innovative because it measures the individual effect of each selected ratio on predicting failure, an approach which he named "traditional". Altman developed a model composed of distinct ratios. The presented model aimed to classify bankrupt and non-bankrupt manufacturing firms by using multi-discriminant analysis (MDA). The ratios used in the model were expected to explain the bankruptcy of a firm by their simultaneous contribution to the model. This approach was necessary because earlier studies on univariate analysis indicated contradictory results due to the different effects of individual ratios. For instance, a firm could be classified as bankrupt because of its high debt ratio, whereas its performance indicators encouraged a non-bankrupt categorization of the same firm. The sample of the study consisted of 66 manufacturing firms divided equally into two groups, and matched in accordance with their asset sizes, as in Beaver's 1966 study. The study indicated the importance of a multivariate analysis of the financial ratios to achieve 95% accuracy in the prediction of bankruptcy within two years, compared to an individual comparison of each financial ratio in a sequence.

Altman's (1973) study employed a linear discriminant analysis in the estimation of the bankruptcy of the US railroads between 1946 and 1969. The results of his model indicated a 97.7% accurate prediction of failure one or two years in advance. A study by Blum (1974) established the "Failing Company Model" using MDA to study the period from 1954 to 1968, using a sample of 115 failed and 115 non-failed industrial firms, and a model consisting of 12 variables. The failed firms were chosen as having a liability over one million dollars, while the matching criteria included the industry, sales, number of employees, and fiscal year. The results of the study indicated the power of the MDA analysis, which produced a 94% accurate classification of failed and non-failed firms in the year before failure.

The accuracy levels decreased as the period to failure increased: 80% for two to three years before failure, and 70% for the third year before failure: Nevertheless, Blum's results highlighted the robustness of the MDA results.

Altman *et al*.'s (1979) study on Brazilian firms in the textile, furniture, pulp and paper, plastics, and metallurgy industries, retail stores, and other firms considered whether problematic and non-problematic firms could be differentiated in advance through linear discriminant analysis. The period of the study spanned from January 1975 to June 1977. The accuracy level of the model was 88% for the 58 firms considered, indicating that the information content of an emerging market is important.

In his 1980 study, Ohlson represented his early prediction model using a logistic regression method. Ohlson's research differed from previous studies in terms of the statistical model and the chosen sample. The conditional logit model, a maximum likelihood estimator, was used to analyze the data, which included 105 bankrupt firms and 2058 non-bankrupt firms from 1970 to 1976. The sample was also unique; rather than Moody's Manual, the year-end 10-K financial statements were used to select bankrupt firms; these have the potential to indicate whether a company will fall into bankruptcy before or after the release of the related data to public. In other words, the timing of bankruptcy could be captured more effectively than in studies using a given sample of bankrupt firms. The predictions of bankruptcy within one year were also improved compared to previous studies, with an accuracy of 96.12%.

Taffler (1983) explored the predictability of the bankruptcy of UK firms using a linear discriminant analysis. The sample was composed of 46 publicly listed firms from 1969 to 1976. The model sample was matched for failed and non-failed firms. The accuracy level was 95.7% for the bankrupt firms and 100% for the non-bankrupt firms. The models have changed over time, and statistical approaches other than MDA have been used.

In 1984, Zmijewski applied a probit analysis to 129 bankrupt industrial firms from a total of 2241 firms in the NYSE and AMEX. Zmijewski found that accuracy levels for the sample varied according to different approaches, including matched sample, non-matched sample, and weighted exogenous sample maximum likelihood (WESML) probit analyses. The accuracy of this study changed depending on the sample. The results for the accuracy of classification for matched sample were 92.5% for failed firms and 100% for non-failed firms, whereas the accuracy of the classification decreased when the sample was non-matched; the accuracy levels were 62.5% for failed firms and 99.5% for non-failed firms. The results of using WESML further decreased the accuracy for failed firms to 52.5% for the matched sample and to 42.5% for the non-matched sample.

Bhatia (1988) measured the classification accuracy of 18 distressed firms and 18 non-distressed firms with an MDA model for the 1976-1985 period, and found 87% accuracy for predicting type I errors and 86.6% for type II. Another study by Ramana *et al*. (2012) in India, using Altman's Z score, indicated that for the 2001-2010 period, the classification accuracy of the MDA model represented the estimation results for three cement companies.

Bidin's (1988) study represents the evaluation of companies owned by an investment trust fund of the government of Malaysia. The related firms in the portfolio of the entity were measured by a multivariate discriminant analysis distress model that was primarily focused on the manufacturing, transportation and service sectors. The sample was composed of 21 distressed companies and an unknown number of non-distressed companies. It was indicated that the government used the revised version of the same model to analyze data covering the period until 1997. Another study was conducted in Singapore in 1981 using multi-discriminant analysis to consider 24 failed and 21 non-failed firms for the period from 1975 to 1983. The companies in the sample were manufacturing and commercial firms. 77.3% accuracy was achieved in the prediction of a type I error one year prior to the failure, and 93.5% for a type II error.

Gilbert *et al*. (1990) measured the prediction accuracy of a logistic regression model and multivariate discriminant analysis for non-financial US firms to determine which method was more accurate. To determine the prediction results of the two models, the sample was divided into two groups, each separated into two sub-groups. The variables were chosen from the models of Casey and Bartczak (1985) and Altman (1968). The sub-samples were composed of bankrupt-to-non-bankrupt, and bankrupt-to-distressed firms to determine the accuracy of the classifications. A logistic regression model was used to measure the separation of bankrupt firms from non-bankrupt ones, and the Altman model was used to measure the separation of bankrupt firms from distressed ones. Bankrupt and non-bankrupt group data were collected through Compustat Annual Industrial or Research Files. The bankrupt group was composed of 76 firms, and the non-bankrupt group included 304 firms. It was found that the accuracy level decreased when the model was processed for the classification of bankrupt and distressed firms. The study highlighted the poor performance of the logistic regression model in the classification of bankrupt and distressed firms.

Shumway (2001) established a dynamic model, a binary logit model, to calculate the financial bankruptcy of firms. The sample was composed of 300 bankrupt firms for the period from 1962 to 1992. Bankrupt firms were defined as those bankrupt within 5 years of delisting. Shumway differentiated his model from the others in the literature through the calculation of firms' trading years, in order to reduce the loss of firms from the sample over time. His simple hazard model was a type of survival analysis that allowed the inclusion of market-driven variables in his model to predict corporate bankruptcy. To emphasize the accuracy power of his model, he adjusted Altman's and Zmijewski's models by adding new market-driven variables. There were two model estimates: one consisted only of market-driven variables, while the other included accounting variables, as a result of considering the studies of Altman and Zmijewski. Altman's and Zmijewski's models were improved through the inclusion of the trading age of the corporations, and following this development, the market models were considered in the sample used by Shumway for the years 1962-1992. Shumway's emphasis was that Altman's model decreased the number of observations, and working capital divided by the total assets variable decreased simply due to the omission of the variables, which also deteriorated the results of statistical significance tests. The model developed another point of view on risk adjustment over the life of the corporations in the sample, by correcting the risk at distinct periods. The model developed through accounting- and market-based variables indicated the most accurate results, as emphasized by Shumway's 95% accuracy rate for market-driven variables.

Ugurlu (2006) measured the prediction accuracy of Turkish manufacturing firms between 1996 and 2003 with multi-discriminant and logit analysis. The logistic regression was found to be more accurate, and the model results indicated that logit regression results provided 95.6% accuracy for the overall fit of the model. The model classified non-failed and failed firms with 97.5% and 91.4% accuracy, respectively. Moreover, the predictive results of the logit model were 94.3%, 91% and 87.1% accuracy from year one to year four, respectively.

Xie *et al*. (2011) compared the accuracy results of MDA and Support Vector Machine (SVM) estimation methods for Chinese listed companies. The study considered the classification of ongoing firms over financial, macroeconomic and internal governance ratios in the prediction model and concluded that SVM provided improved accuracy rates for distress classification due to its non-parametric structure conceding the sample distribution as non-linear. A study by Bauer and Agarwal (2014) indicated that Taffler (1983) Z-score and hazard models have the clear bankruptcy classification abilities while hazard models were more cost efficient in terms of their prediction performances over risk weighted assets.

Although bankruptcy estimation model studies have a long history in the accounting and finance literature, studies so far have primarily focused on developing new models using different financial statement variables. Moreover, the literature has generally focused on the accuracy levels of the bankruptcy models applied to bankrupt firms in a matched sample of bankrupt to non-bankrupt firms, even though the models can be applied to distinguish financially healthy firms from the firms that were not bankrupt but were called financially weak and distressed firms by Altman (1968) and Gilbert *et al*. (1990).

3. Research design

Following the models of Altman (1968), Ohlson (1980), Zmijewski (1984), Taffler (1983), and Shumway (2001), the current study considers the original coefficients to examine whether these models provide significantly accurate prediction results through their original coefficients for publicly listed Turkish industrial firms. Then, each of the models is re-estimated and the coefficients are calculated to analyze if the prediction accuracies are improved. Additionally, the models are processed in their original and re-estimated versions, considering the financial crisis of 2008 as a cut-off year for the comparison of prediction results. The comparison analysis is conducted through the use of accuracy rates calculated by the rate of the correct classification of the financially distressed or non-distressed firm observations to total sample observations.

3.1 Data

The study sample is composed of publicly listed industrial firms for the validity of the prediction results, as the models are derived from publicly listed industrial firms of the developed markets. In addition to that the sample is also plausible due to Turkey's well regulated market structure in comparison to other developing markets and the Borsa Istanbul's high trading volume, the fourth largest among emerging markets.

The study covers the period from 2000 to 2012. The dataset is collected from two reliable sources, the Bloomberg Professional and the Thomson Reuters Eikon data terminals. Besides being an industrial firm, another condition is to be listed in the stock market between the years 2000 to 2012, otherwise the dataset would have been failed in terms of consistency. The maximum industrial firm number is observed between the years 2000 and 2012. Therefore the dataset is composed of 45 publicly listed industrial firms, with 585 observations for the full sample in the Borsa Istanbul with 140 financially distressed (FD) and 445 non-distressed firms. The sample period also satisfies the relative observation distribution for pre and post financial crisis which enables the analysis of the related prediction accuracies. Industrial firms were selected in order to sustain the validity of the models established for this type of firms in developed markets. The sample was also divided into two sub-samples in order to analyze the models' level of accuracy before and after the financial crisis of 2008, with 360 pre-crisis and 225 post-crisis observations. After the data collection, outliers within the dataset were extracted based on a 95% confidence level. Two important conditions for inclusion in the sample were to be an industrial firm and to be listed in the stock market between 2000 and 2012; this was to prevent the dataset being biased and failing in terms of consistency.

After the data collection, descriptive statistics were produced to check whether the variables were appropriate for estimation. Table 1 represents the descriptive statistics for all firms.

Table 1. Descriptive statistics for the variables

	Mean	Standard Error	Median	Mode	Standard Deviation	Kurtosis	Skewness
CL/CA	1.51	0.10	1.16	0.96	2.49	6.20	340.69
WC/TA	0.01	0.03	0.07	2.53	0.71	0.51	116.88
RE/TA	0.00	0.03	0.11	2.38	0.74	0.55	105.53
$EBIT/TA$	0.08	0.00	0.07	0.09	0.12	0.01	11.79
$SALES/TA$	0.98	0.03	0.90	0.91	0.69	0.48	5.08
MVE/TL	4.57	0.50	1.15	0.00	12.85	165.18	48.91
TL/TA	0.60	0.03	0.53	0.45	0.69	0.47	140.69
$OENEG$	0.04	0.01	0.00	0.00	0.20	0.04	19.19
ROA	5.68	0.54	5.20	28.61	14.02	196.43	18.06
$CHIN$	0.07	0.02	0.05	0.00	0.53	0.28	0.16
$SIZE$	3.07	0.01	3.02	3.04	0.29	0.08	0.92
$RETURN$	0.06	0.02	0.00	0.00	0.47	0.22	6.48
$SIGMA$	0.04	0.00	0.03	0.04	0.02	0.00	10.17
PBT/ACL	0.32	0.04	0.19	0.03	1.02	1.05	35.44
CA/TL	1.08	0.05	0.84	0.21	1.30	1.70	143.48
CA/CL	0.42	0.03	0.35	0.15	0.66	0.43	174.71
$(CA - INV - CL)/(SALES - NIBT + DEPR)$	2.88	1.10	0.13	29.80	28.61	818.58	217.75

The descriptive statistics show that the median values for all variables are close to the mean values, indicating that the data seem to be evenly distributed around the mean.

3.2. Methodology

Altman's Methodology

Estimation Method: Discriminant Analysis

Discriminant analysis allows the simultaneous examination of differences between two or more groups of objects with respect to several variables. In the social sciences, there are a wide variety of situations in which this technique may be useful (Huberty & Olejnik, 2006). Discriminant analysis aims to determine:

1. Which variables within the equation, if any, are effective in predicting the ultimate outcome
2. How these variables might be combined into a mathematical equation to predict the most likely outcome, and
3. The accuracy of the derived equation.

The basic prerequisites are the existence of two or more groups, which are presumed to differ in terms of several variables, and that those variables can be measured at the interval level or ratio level. Discriminant analysis will then enable the analysis of the differences between the groups and provide a means to assign a particular case into the group it most closely resembles.

Ohlson's Methodology

Estimation Method: Logit Model

The logit model is similar to the probit model because it is a binary model, but it has a number of features that make it more convenient than the probit model (Gujarati, 2004). For the logit model, the function $F(x)$ is the logistic function

$$\Lambda(x) \equiv (1 + e^{-x})^{-1} = \frac{e^x}{1 + e^x}$$

which has a first derivative

$$\lambda(x) \equiv \frac{e^x}{(1 + e^x)^2} = \Lambda(x)\Lambda(-x)$$

The second equality will later prove to be useful. The logit model is most easily derived by assuming that

$$\log\left(\frac{P_t}{1 - P_t}\right) = X_t\beta$$

which says that the logarithm of the odds is equal to $X_t\beta$. Solving for P_t,

$$P_t = \frac{\exp(X_t\beta)}{1 + \exp(X_t\beta)} = (1 + \exp(-X_t\beta))^{-1} = \Lambda(X_t\beta)$$

It is also possible to derive the logit model from a latent variable model, but with errors that follow extreme rather than normal value distributions; see, among others, Domencich and McFadden (1975), McFadden (1984), and Train (1986).

Shumway's Methodology

Estimation Method: Simple Hazard Model

The logit and hazard models have the same likelihood functions, and therefore the same asymptotic variance-covariance matrices (Amemiya, 1985). However, the test statistics produced by a logit program is incorrect for the hazard model because the assumption is that the number of independent observations used to estimate the model is equal to the number of years for each firm within the data sample. Calculating correct test statistics requires an adjustment of the sample size assumed by the logit program to account for the lack of independence between firm-year observations.

Zmijewski's Methodology

Estimation Method: Probit Model

For the probit model, the transformation function $F(x)$ is the cumulative standard normal distribution function

$$\Phi(x) \equiv \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}X^2\right) dX$$

Because $\Phi(x)$ is a cumulative distribution function, it automatically satisfies the condition above. The probit model can be written as

$$P_t \equiv E(y_t|\Omega_t) = \Phi(X_t\beta)$$

Although there exists no closed-form expression for $\Phi(x)$, the variable is easily evaluated numerically, and its first derivative is simply the standard normal density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)$$

The probit model can be derived from a model that involves an unobserved or latent variable y_t^*.

$$y_t^* = X_t\beta + u_t, \qquad u_t \sim NID(0,1)$$

Only the sign of y_t^*, is observed which determines the value of the observed binary variable y_t according to the relationship

$$y_t = 1 \; if \; y_t^* > 0 \;\; and \;\; y_t = 0 \; if \; y_t^* \leq 0$$

The variance of u_t is normalized for unity. If u_t actually had some other variance, such as σ^2, dividing y_t^*, β, and u_t by σ would yield a model observationally identical to the one with is started.

Now it can be asked what the probability is that $y_t = 1$. Some straightforward manipulations yield

$$\begin{aligned} \Pr(y_t = 1) &= pr(y_t^* > 0) = \Pr(X_t\beta + u_t > 0) = 1 - \Pr(u_t \leq -X_t\beta) \\ &= 1 - \Phi(X_t\beta) = \Phi(X_t\beta) \end{aligned}$$

The last equality exploits the fact that the standard normal density function is symmetric around zero. The final result, $\Phi(X_t\beta)$, is the probability that would be obtained by letting $\Phi(.)$ play the role of $F(.)$. Thus, the probit model is derived from the latent variable model. One of the advantages of the probit model is that it can be created in this way (Baltagi, 2008).

Taffler's Methodology

Estimation Method: Linear Discriminant Analysis

The analysis of variance is closely related to LDA because it attempts to express one dependent variable as a linear combination of other features or measurements. The difference between the analysis of variance and LDA is that whereas the former uses categorically independent variables with a continuous dependent variable, the latter uses continuous independent variables with a categorically dependent variable. In terms of explaining a categorical variable through continuous independent variables, LDA is closer to logistic regression and probit regression. An important feature common to logistic regression and probit regression is the lack of requirement for normally distributed independent variables, in contrast to LDA, which strictly requires these variables (Huberty & Olejnik, 2006).

The training set is the set of samples that includes the set of observations $\vec{x}$ for each sample of an event with known as class y. Finding a good predictor of the class y of any sample of the same distribution given only an observation $\vec{x}$ refers to the classification problem.

LDA assumes that the conditional probability density functions $p(\vec{x}|y=0)$ and $p(\vec{x}|y=1)$ are normally distributed with mean and covariance parameters $(\vec{\mu}, \Sigma_{y=1})$ and $(\vec{\mu}, \Sigma_{y=1})$ to solve the problem. The Bayes optimal solution is to predict second-class points if the log of the likelihood ratios falls below a threshold T under the following assumption.

$$(\vec{x}-\vec{\mu}_0)^T \Sigma_{y=0}^{-1}(\vec{x}-\vec{\mu}_0) + \ln|\Sigma_{y=0}| - (\vec{x}-\vec{\mu}_1)^T \Sigma_{y=0}^{-1}(\vec{x}-\vec{\mu}_1) - \ln|\Sigma_{y=1}| < T$$

The resulting classifier is referred to as quadratic discriminant analysis without any other assumption. By generating the co-variances that have full rank, this methodology achieves the simplification of the homoscedasticity assumption. Therefore, the above decision criterion becomes a threshold on the scalar product.

$$\vec{w} \cdot \vec{x} > c$$

for some threshold constant c, where

$$\vec{w} \propto (\Sigma_0 + \Sigma_1)^{-1}(\vec{\mu}_1 - \vec{\mu}_0)$$

$$c = \vec{w} \cdot \frac{1}{2}(\vec{\mu}_0 + \vec{\mu}_1) = \frac{1}{2}\vec{\mu}_1^T \Sigma^{-1}\vec{\mu}_1 - \frac{1}{2}\vec{\mu}_0^T \Sigma^{-1}\vec{\mu}_0$$

The result indicates that the criterion of an input $\vec{x}$ being in a class y is purely a function of this linear combination of the known observations.

4. Empirical results

The results are obtained by considering each model's variables over the original coefficients, and the coefficients re-estimated. The five models that are analyzed for Turkish industrial firms through financial distress levels, depending on two years of financial distress, and the prediction period, which covers the period from one to five years prior to the distress are considered. After the re-estimation process, the updated models are represented below and comparative coefficients in Table A.1.

Altman	$(1.42)WC/TA + (0.31)RE/TA + (8.05)EBIT/TA + (0.00)MVE/TL$ $-(0.02)SALES/TA$
Ohlson	$-(1.34) - (0.21)SIZE - (0.45)TL/TA - (2.13)WC/TA + (0.01)CL/CA$ $-(0.30)ROA + (0.00)OCF/TA - (1.06)OENEG + (1.09)CHIN$
Shumway	$(2.81) - (1.44)SIZE + (0.02)RETURN - (0.25)ROA - (6.99)SIGMA$ $+(0.26)TL/TA$
Zmijewski	$-(0.60) + (0.001)CA/CL - (0.13)ROA - (0.06)TL/TA$
Taffler	$-(0.87) - \frac{(0.09)PBT}{ACL} + \frac{(0.009)CA}{TL} + \frac{(2.59)CL}{TA} - (0.03)\frac{CA-INV-CL}{SALES-NIBT+DEPR}$

The results indicate that the most accurate prediction results for financial distress one year in advance for Turkish manufacturing firms are derived from the original Zmijewski model (Table 2). The only model that results in an improved level of prediction accuracy when re-estimated is the Ohlson model in advance of one year before financial distress. One of the reasons for this improvement could be that the model works better with a more recent data sample, as indicated by (Begley *et al.*, 1996).

Taffler model gives the second most accurate results through its original coefficients, and its prediction ability decreases when the model is re-estimated. This result is also compatible with those of Taffler and Agarwal (2007), who aim to measure the predictive power of Taffler's (1983) model and find that the model is more effective when applied with the original coefficients for the UK sample which concludes that the longevity of the model does not mean that it is outdated. Likewise, Shumway's model prediction accuracy also falls when the model is re-estimated. Although Shumway's model differentiates itself from the other models by having market-based variables, for Turkish industrial firms, its prediction accuracy is lower than all other models, with the exception of Altman's model, which has the lowest accuracy level of all models.

Table 2 Comparison of accuracy levels of each model

Model	*Lag*	*Original*			*Re-estimated*		
		t_{γ}	t_{ϑ}	t_{φ}	t_{γ}	t_{ϑ}	t_{φ}
Altman	[1]	32.5%	35.9%	28.4%	33.5%	39.3%	26.7%
	[2]	30.4%	34.7%	26.2%	32.2%	38.2%	26.2%
	[3]	30.1%	36.1%	25.3%	31.9%	38.9%	26.2%
	[4]	28.3%	37.0%	23.1%	30.3%	40.0%	24.4%
	[5]	23.4%	31.1%	20.4%	24.7%	34.4%	20.8%
Ohlson	[1]	75.8%	70.7%	81.8%	80.6%	76.3%	85.8%
	[2]	77.3%	72.9%	81.8%	77.8%	72.9%	82.7%
	[3]	77.8%	72.8%	81.8%	77.0%	73.3%	80.8%
	[4]	78.9%	74.1%	81.8%	76.1%	69.6%	80.8%
	[5]	76.5%	71.2%	81.8%	75.2%	67.2%	80.8%
Shumway	[1]	80.6%	74.4%	88.0%	78.2%	81.1%	74.7%
	[2]	77.1%	72.0%	82.2%	68.9%	72.4%	65.3%
	[3]	76.5%	72.8%	79.6%	64.4%	70.0%	60.0%
	[4]	75.6%	69.6%	79.1%	62.5%	65.9%	60.4%
	[5]	74.1%	66.4%	77.2%	61.0%	62.1%	59.2%
Zmijewski	[1]	85.9%	85.9%	85.8%	80.0%	74.4%	86.7%
	[2]	81.2%	80.7%	81.8%	80.6%	74.4%	88.0%
	[3]	72.3%	74.4%	70.7%	77.0%	72.8%	80.4%
	[4]	69.4%	71.1%	68.4%	75.8%	69.6%	79.6%
	[5]	67.5%	69.8%	69.3%	72.9%	67.8%	78.2%
Taffler	[1]	80.6%	78.1%	83.6%	73.3%	68.1%	79.6%
	[2]	76.7%	75.6%	77.8%	74.9%	70.2%	79.6%
	[3]	75.8%	76.1%	75.6%	75.6%	70.0%	80.0%
	[4]	76.1%	75.6%	76.4%	76.4%	71.1%	79.6%
	[5]	71.5%	72.1%	74.7%	77.3%	71.6%	79.8%

Notes: t_{γ}, t_{ϑ}, and t_{φ} are the period identifiers when the auxiliary accuracy results are calculated for the full period, pre-crisis period, and post-crisis period, respectively.

When the sub-samples are considered in terms of pre- and post-financial crisis results, the models provide greater accuracy with their original coefficients, with an average increase in accuracy of 4.5% after the financial crisis of 2008, and the re-estimated model results show a corresponding increase of 2.9%. As for the results for the full sample, no improvement is shown in accuracy for the re-estimated results in relation to the original coefficients when the models are compared by considering the sub-samples of the periods before and after the financial crisis of 2008.

In order to measure the validity of the results, the matched hold out sample of 100 FD and 100 Non-FD firms are selected. The results indicate that four of the models have the predictive accuracy for Turkish publicly listed industrial firms (Table

A.2). Additionally, the type I and type II error rates (Table A.3) emphasize that Shumway,Zmijeski and Taffler models provide improved type I error classification whereas each model other than Ohlson provides deteriorated type II error results when re-estimated.

5. Conclusions

The current study analyzes the generalizability of financial distress estimation models in an emerging economy derived for developed markets. The results emphasize that all models, with the exception of Altman (1968), are generalizable and capable of classifying firms in an emerging market as distressed or not distressed, while also establishing a basis for further prediction models. Because the ratio selection process has no theoretical base, and researchers may use many different ratios, generalizable models have the potential to create a benchmark which could be used when establishing new models for prediction accuracy. On the other hand, the models' ability to classify ongoing industrial firms, rather than bankrupt firms provides practitioners with an important decision-making tool, while indicating which model provides the best accuracy for Turkish industrial market.

The results also emphasize that Altman (1968) model is not able to process distress classification for ongoing firms. However, the remaining models are appropriate for predicting the distress of listed industrial firms in Turkey through the original coefficients for ongoing firms, with an average accurate prediction rate of 81%. The re-estimation results of the models indicate that the re-estimation improves the Ohlson (1980) model results by 4.8%, but has a negative effect on all other models. The models' prediction results over original coefficients give better results after the financial crisis of 2008; in contrast, the re-estimated model predictions after the financial crisis period are mixed, and do not allow a precise interpretation to be obtained. It can therefore be concluded that the distress prediction models derived for developed markets provide robust prediction results, and are capable of classifying ongoing firms in an emerging market sample. Where the data is available, further research may be able to shed light on the prediction results of these models for other emerging market countries.

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Appendix

Table A.1 Re-estimated coefficients of each model

X	Altman		Ohlson		Shumway	
	β	$\hat{\beta}$	β	$\hat{\beta}$	β	$\hat{\beta}$
X_1	0.012	1.419	-0.410	-0.207	-0.480	-1.439
X_2	0.014	0.312	6.030	-0.451	-1.810	0.022
X_3	0.033	8.050	-1.430	-2.129	-1.980	-0.253
X_4	0.006	-0.022	0.070	0.002	5.790	-6.994
X_5	0.999	0.003	-2.370	1.086	3.590	0.260
X_6			-1.830	-0.297		
X_7			-1.720	0.000		
X_8			- 0.520	-1.062		
	Zmijewski		**Taffler**			
X_1	β	$\hat{\beta}$	β	$\hat{\beta}$		
X_2	0.004	0.001	12.18	-0.089		
X_3	-4.500	-0.126	2.500	0.009		
X_4	5.700	-0.059	-10.68	2.587		
X_5			0.030	-0.032		

Notes: X represents the independent variables of each model, β stands for the original coefficients, and $\hat{\beta}$ indicates re-estimated coefficients.

Table A.2 Comparison of accuracy levels of each model for holdout sample

	Lag	*Model*			
		Zmijewski	Taffler	Shumway	Ohlson
Original	[1]	88.8%	77.7%	72.1%	63.1%
	[2]	83.5%	77.8%	72.2%	65.2%
	[3]	74.3%	72.9%	71.4%	67.1%
	[4]	73.0%	73.8%	71.3%	69.7%
	[5]	74.3%	72.9%	71.4%	70.8%
Re-estimated	[1]	69.3%	62.6%	77.1%	70.9%
	[2]	70.9%	64.6%	77.8%	70.9%
	[3]	71.4%	66.4%	73.6%	68.6%
	[4]	71.3%	68.9%	73.8%	69.7%
	[5]	71.4%	66.4%	73.6%	68.9%

Table A.3 Comparison of Type I and Type II results

Model	*Lag*	*Original* e_I	*Original* e_{II}	*Re-estimated* e_I	*Re-estimated* e_{II}
Ohlson	[1]	1.3%	17.9%	5.9%	13.3%
	[2]	0.6%	17.4%	5.5%	12.5%
	[3]	1.7%	15.4%	5.9%	11.1%
	[4]	2.8%	13.0%	6.3%	9.6%
	[5]	3.8%	11.1%	7.1%	7.9%
Shumway	[1]	5.2%	14.0%	2.1%	18.1%
	[2]	4.5%	13.7%	2.2%	16.2%
	[3]	8.1%	12.1%	4.4%	14.4%
	[4]	11.3%	10.6%	6.5%	12.6%
	[5]	8.1%	12.1%	4.4%	14.4%
Zmijewski	[1]	17.1%	1.7%	0.5%	19.5%
	[2]	15.4%	4.8%	2.0%	16.9%
	[3]	22.5%	4.4%	3.9%	14.9%
	[4]	25.8%	4.4%	5.5%	12.8%
	[5]	22.5%	4.4%	3.9%	14.9%
Taffler	[1]	7.6%	11.6%	0.9%	20.5%
	[2]	8.1%	11.5%	2.8%	17.4%
	[3]	11.5%	10.4%	3.9%	15.4%
	[4]	13.2%	8.7%	5.1%	13.0%
	[5]	11.5%	10.4%	3.9%	15.4%

Notes: e_I, and e_{II} represents Type I and Type II errors respectively.