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Abstract: α ("Alpha") has symbolic importance on the investments side of finance. That is, a fundamental pillar of modern finance theory is the risk-return relation, and traditionally alpha is taken to represent the degree of "mispricing" in asset returns. But, such an interpretation is not always appropriate – seemingly paradoxically, for certain specific setups alpha embodies pricing information. In this paper, I explain and illustrate the distinguishing circumstances between these two diametrically opposed cases.

Keywords: Alpha; Beta; empirical asset pricing; mispricing information; pricing information

JEL codes: G12

1. Introduction

In the academic discipline of finance, the genesis of modern asset pricing theory begins with Markowitz (1952, 1959) in formalizing our understanding of diversification and portfolio theory. The CAPM followed, thanks to Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM states that there is a positive and linear relationship between (standardized covariance) risk, i.e. β ("beta"), and expected return. Indeed, for all students of modern finance β has become synonymous with the CAPM. Nevertheless, the validity/usefulness of the CAPM (and, hence, beta) has been mired in an extensive debate across the literature.[1] Indeed, many alternative asset pricing models/benchmarks have been proposed and used in various contexts including the assessment of managed fund performance and event studies.[2] For example, in their latest foray, Fama and French (2014) propose a new five-factor model which augments their three-factor model with an investment factor and a profitability factor.[3]

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Asset pricing models/benchmarks can and are used in a variety of contexts and quite often the associated empirical setup is cleverly designed so that insightful information is captured by (or "forced" into) the intercept term, a ("alpha"). In this paper, I explain and illustrate the role of alpha in empirical asset pricing – where (depending on the specific nature of the setup) a non-zero alpha can represent "mispricing" or pricing information.

This paper adopts the following structure. Section 2 outlines and discusses the zero alpha testing setup. Section 3 explores the role of alpha in testing cross-equation restrictions implied by the Sharpe-Linter-Mossin CAPM. Section 4 explores the role of alpha in testing cross-equation restrictions implied by other asset pricing models. Finally, in Section 5 I offer some brief concluding remarks.

2. Basic Zero Alpha Test

2.1 Four-step Theme of Analysis

As we shall soon see, a recurring "theme" quickly emerges – built around a four-pronged research strategy. In Step 1 we identify the basic asset pricing model of interest and manipulate it into a form that is most "conducive" to performing the empirical test (one that has a multivariate/systems approach in mind). In Step 2 we specify the empirical counterpart to the asset pricing model in step 1 (which might involve some critical design choices to best exploit the data/tools available for the test). In Step 3 we then take statistical expectations through the empirical specification identified in Step 2. In Step 4 we "match up" terms/variables from the asset pricing model versus its (expected) empirical counterpart, such that the testable hypothesis turns out to be of the general form "α = …". The current section traces through these four steps in the context of the simplest and most well-known asset pricing model, the Sharpe-Lintner-Mossin CAPM.

2.2 Sharpe-Linter-Mossin CAPM

The Sharpe-Lintner-Mossin CAPM states that there is a positive and linear relationship between (standardised covariance) risk, β_i, and expected return ($E(R_i)$), for asset i:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \quad \text{(T1)}$$

where R_f is the risk-free rate of return and $E(R_m)$ is the expected return on the market portfolio.[4] By subtracting the risk-free rate from both sides and defining excess returns (relative to R_f) with lower case "r", we can re-express the CAPM as:

$$E(r_i) = \beta_i E(r_m) \quad \text{(T2)}$$

The empirical (ex post) counterpart to the CAPM is the market model which, expressed in excess returns form, is given by:

$$r_{it} = \alpha_i + b_i r_{mt} + \varepsilon_{it} \quad \text{(E1)}$$

where r_{it} (r_{mt}) is the realized excess return on asset i (market index) in period t. The sample has T time-series observations. If we take (sample) expectations through equation (E1), noting that by definition the error term has a mean of zero:

$$\bar{r}_i = \alpha_i + b_i \bar{r}_m \quad \text{(E2)}$$

where $\bar{r}_i$ ($\bar{r}_m$) is the sample mean excess return on asset i (market index).

Comparing equation (T2) with equation (E2) we see three matched elements: (a) the LHS test asset i return variables match in terms of economic theory, $E(R_i)$, and a given sample, $\bar{r}_i$; (b) the RHS market return variables match in terms of economic theory, $E(R_m)$, and a given sample, $\bar{r}_m$; and (c) given (b), the estimated loading on the market in the market model, b_i, is the empirical counterpart of the theoretical beta in the CAPM. This leaves just one outstanding component in equation (E2), namely α_i, for which there is no counterpart to match against in the CAPM. In other words, for the CAPM (and the associated theory underlying its construction) to be "true" i.e. for the CAPM to be supported by the data; then alpha must be **zero**.[5] Indeed, this identifies the null hypothesis, H1, for the CAPM in this setting:

$$H_0: \alpha_i = 0 \quad \text{(H1)}$$

As expressed above, H1 is tested for a single test asset (i.e. a single LHS variable at a time), but such a test is quite weak since joint information across assets is ignored (relative to the proposed pricing factors).[6] A more powerful version is to apply a joint-test simultaneously across many test assets (say, "I" test assets) – a so-called "multivariate" test, for which the null hypothesis, H2, becomes:

$$H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_I = 0 \quad \text{(H2)}$$

Examples of this type of test can be found in Connor & Korajczyk (1988), Faff (1992) and Faff & Lau (1997). Connor & Korajczyk (1988) tests H2 in the US using monthly data, over the period 1964 to 1983, whereas Faff (1992) performs similar tests on an Australian sample using monthly data, over the period 1974 to 1987. The chosen test assets are individual stocks and size decile portfolios, respectively. Faff (1992) tests an unconditional zero alpha restriction, as well as conditional tests looking at monthly seasonal alphas (notably January, July and August). Faff & Lau (1997) apply the test to an Australian sample using monthly data, over the period 1974 to 1994. In this latter case, the chosen test assets are 24 industry portfolios.

This testing approach and hypothesis is readily extended to multi-factor models. The simple principle involves matching on k factors on (a) the RHS in terms of the theoretical factor loadings in product with the associated expected factor premia and (b) the product of the estimated factor loadings with the associated sample mean of factor returns. For example, Faff (1992) also applies the unconditional and conditional tests as mentioned above, to a k-factor APT. Indeed these APT tests are the main focus of that paper – both five and ten-factor APT specifications are investigated.

3. Testing Cross-equation Restrictions in the Sharpe-Linter-Mossin CAPM

In the context of empirical asset pricing research, depending on the setup, alpha can either represent "mispricing" or pricing information. In the preceding section, the simple "zero alpha" test is the primary example of the former – given that empirical setup, financial economic theory predicts no role for a non-zero alpha term and so if the data do tell us that alpha is important, it reflects "mispricing" information relative to the asset pricing model being tested.[7]

The alternative approach of setting up a null hypothesis that embodies the asset pricing information can most readily be illustrated in the context of the Sharpe-Lintner-Mossin CAPM. As before, the market model is recognized as the empirical counterpart – but in this case, it is expressed in raw returns form (i.e. realized unadjusted returns), given by:

$$R_{it} = \alpha_i + b_i R_{mt} + \varepsilon_{it} \qquad \text{(E3)}$$

where R_{it} (R_{mt}) is the realised raw return on asset i (market index) in period t. Again, we take (sample) expectations through equation (E3) and noting that, by definition, the error term has a mean of zero:

$$\bar{R}_i = \alpha_i + b_i \bar{R}_m \qquad \text{(E4)}$$

One further piece of minor manipulation is required to the theoretical model, before we can proceed to the "matching" of terms step. Specifically, the bracketed terms on the RHS of the CAPM, as expressed in equation (T1), can be expanded:

$$E(R_i) = R_f + \beta_i E(R_m) - \beta_i R_f \qquad \text{(T3)}$$

and the terms involving R_f collected together:

$$E(R_i) = R_f(1 - \beta_i) + \beta_i E(R_m) \qquad \text{(T4)}$$

Comparing equation (T4) (i.e. the economic model) with equation (E4) (i.e. the expected form of the sample counterpart model), we match off elements: (a) the LHS test asset i raw return variables match in terms of economic theory, $E(R_i)$, and a given sample, $\bar{R}_i$; (b) the RHS market return variables match in terms of economic theory, $E(R_m)$, and a given sample, $\bar{R}_m$; and (c) given (b), the estimated loading on the market in the market model, b_i, is the empirical counterpart of beta in the CAPM. This leaves one outstanding component in equation (E4), α_i, which by default, on average must be equal to the remaining term in the CAPM in equation (T4). In other words, if the CAPM (and the associated theory underlying its construction) is "true" i.e. it is supported by the data, then the null hypothesis for the CAPM in this setting becomes:

$$H_0: \alpha_i = R_f\,(1 - \beta_i) \qquad \text{(H3)}$$

There are three things to note about hypothesis H3 in comparison to hypothesis H1. First, unlike H1 (& indeed, H2) which is setup such that alpha captures mispricing information, in H3 alpha captures the underlying pricing content of the model. That is, we are looking for a very specific relation between the two key pricing parameters underlying the CAPM, namely the risk-free rate of return and beta (i.e. systematic risk). Second, the test of H3 is infeasible with regard to using one test asset in isolation – the market model can only produce estimates of two independent parameters, while the hypothesis requires identification of three separate parameters. The solution is straight forward: perform the test in its multivariate form, thereby seeking to see if the data support the imposition of this cross-equation restriction. Thus, the testable (multivariate) version of the hypothesis is (given "I" test assets):

$$H_0: \alpha_1 = R_f\,(1\text{-}\beta_1);\ \alpha_2 = R_f\,(1\text{-}\beta_2);\ \ldots \quad \alpha_I = R_f\,(1\text{-}\beta_I) \qquad \text{(H4)}$$

Further, recognizing that the risk-free rate parameter is a common link in the joint test of H4, some simple manipulation delivers an equivalent version that clearly shows the cross-equation restriction:

$$H_0: R_f = [\alpha_1/(1\text{-}\beta_1)] = [\alpha_2/(1\text{-}\beta_2)] = \ldots = [\alpha_I/(1\text{-}\beta_I)] \qquad \text{(H4)}$$

This version of H4 shows that the CAPM will be supported if the data "happily" conform to the restriction that a specific ratio form of pricing parameters (i.e. the ratio of alpha to one minus its associated beta) are equal across all test assets and, in turn, are all jointly equal to a positive fixed quantity, interpreted here to be the risk-free rate.

The third and final notable observation regarding the multivariate hypothesis of H4 is its non-linear nature. That is, the key pricing parameters mix together multiplicatively – as opposed to having an additive relation.

4. Testing Cross-equation Restrictions in Other Asset Pricing Models

4.1 Black's Zero-beta CAPM

When the risk-free asset does not exist (a more realistic assumption), Black (1972) argues that rational investors can be thought of as forming optimal combinations of the market portfolio and the zero-beta portfolio. Further, he shows that a CAPM equation of very similar form to the Sharpe-Lintner-Mossin CAPM will hold. Specifically, Black's (1972) zero-beta CAPM states that there is a positive and linear relationship between β_i and expected return for asset i, as follows:

$$E(R_i) = R_Z + \beta_i[E(R_m) - R_Z] \quad \text{(T5)}$$

where R_z is the expected rate or return on the zero-beta portfolio. A simple manipulation of this model (analogous to that which moved us from (T1) to (T4) above), gives an equivalent expression for the Black-CAPM:

$$E(R_i) = R_z(1 - \beta_i) + \beta_i E(R_m) \quad \text{(T6)}$$

Following the same line of reasoning as above, we can compare the Black-CAPM in equation (T6), with the raw return version of the empirical counterpart market model (with sample expectations imposed) in equation (E4), repeated below for convenience:

$$\bar{R}_i = \alpha_i + b_i \bar{R}_m \quad \text{(E4)}$$

This delivers a new hypothesis, H5:

$$H_0: \alpha_1 = R_z(1-\beta_1);\ \alpha_2 = R_z(1-\beta_2);\ \ldots \alpha_I = R_z(1-\beta_I) \quad \text{(H5)}$$

An example of this type of testing approach can be found in Faff and Lau (1997).

Alternatively, the Black-CAPM can be re-expressed in excess returns form as:[8]

$$E(r_i) = r_z(1 - \beta_i) + \beta_i E(r_m) \quad \text{(T7)}$$

Comparing the Black-CAPM in equation (T7), with the excess return version of the empirical counterpart market model (with sample expectations imposed) in equation (E2), repeated below for convenience:

$$\bar{r}_i = \alpha_i + b_i \bar{r}_m \quad \text{(E2)}$$

produces hypothesis H6:

$$H_0: \alpha_1 = r_z(1-\beta_1); \alpha_2 = r_z(1-\beta_2); \ldots \alpha_I = r_z(1-\beta_I) \quad \text{(H6)}$$

Note in this case that if the risk-free rate "exists" (i.e. a reasonable proxy can be found by investors), then the Black-CAPM collapses to the S-L CAPM. This means that the excess return on the "zero-beta" asset is zero, and H6 collapses to H2.

4.2 CCAPM

The consumption CAPM (CCAPM), developed by Rubinstein (1976), Lucas (1978) and Breeden (1979), asserts that cross-sectional variation in average returns can be explained by differences in firms' exposure to consumption risk. According to the CCAPM, assets that covary positively with consumption growth are less attractive to investors since they do not offer a hedge in bad times, and therefore, investors should be compensated by a high expected return for such assets.[9] The fundamental form of the model closely resembles the standard CAPM:

$$E(R_i) = \gamma_0 + \gamma_1 \beta_{Ci} \quad \text{(T8)}$$

where β_{Ci} is a "consumption" beta – a measure of systematic risk relative aggregate consumption growth (i.e. standardized covariance of asset returns with consumption growth). The coefficients γ_0 and γ_1 are two pricing parameters: γ_0 is the risk-free return if it exists, otherwise it becomes the expected zero-beta return in a Black world and γ_1 is the consumption risk premium or market price of consumption beta risk (analogous to the market risk premium in the CAPM).

To test this model, we need to express an analogous model to the market model, but now one that adequately describes how (real) returns are generated in terms of the growth in per capita real consumption, C_t, (see Faff, 1998). Thus, the "consumption" model is:

$$R_{it} = \alpha_i + b_i C_t + \varepsilon_{it} \quad \text{(E5)}$$

Following Breeden *et al.* (1989: 250), a test of the CCAPM is simplified if the consumption growth series are (sample) mean-adjusted, so we assume that this is the case for the consumption series specified in equation (E5). Taking sample expectations through (E5) gives:

$$\bar{r}_i = \alpha_i \quad \text{(E6)}$$

Thus, comparing the CCAPM reflected in (T8) with the empirical counterparts in (E5) and (E6), the theoretical restriction, and hence the null hypothesis for the CCAPM is given by:

$$H_0: \alpha_i = \gamma_0 + \gamma_1 \beta_{Ci} \qquad \forall i \qquad \text{(H7)}$$

Again we note that this is a multivariate, non-linear cross-equation restriction – alpha is designed to fully capture the relevant asset pricing information. If the data "comply" to the economic model (i.e. the CCAPM), tests will be unable to reject the theoretical restriction shown in H7.

An example of this type of testing approach can be found in Faff (1998). Faff (1998) applies the test to an Australian sample using monthly and quarterly data, over the period 1974 to 1992. The chosen test assets are 23 industry portfolios.

4.3 Imputation-adjusted CAPM

Brennan (1970) develops a version of the CAPM that incorporates the tax disadvantage of dividends relative to capital gains under a classical tax system. Australia changed from a classical tax system to a dividend imputation system in July 1987. Such a tax change suggests that the relation between beta and return should be more steeply sloped than observed prior to the change, or compared to other classical tax settings (such as the US). Faff *et al.* (2000) test the empirical validity of an imputation-adjusted CAPM (see Wood, 1997), which has the following economically derived form:[10]

$$E(R_i) = R_f - \tau_i + \beta_i [E(R_m) + \tau_m - R_f] \qquad \text{(T9)}$$

where raw returns are total returns ignoring imputation tax credits (i.e. defined as the sum of the two traditional return components: capital gains and dividends) and τ_i (τ_m) is the tax credit yield on security i (the market). In other words this setup captures the fact that investors have three sources of return comprising their total return: (a) capital gains; (b) dividends; and (c) tax credits.

Equation (T9), with some minor algebraic re-arrangement, can be re-written as:

$$E(r_i) = (\beta_i \tau_m - \tau_i) + \beta_i E(r_m) \qquad \text{(T10)}$$

Comparing the imputation-adjusted CAPM in equation (T10), with the excess return version of the empirical counterpart market model (with sample expectations imposed) in equation (E2), repeated below for convenience:

$$\bar{r}_i = \alpha_i + b_i \bar{r}_m \qquad \text{(E2)}$$

produces hypothesis H8:

$$H_0: \alpha_i = \beta_i \tau_m - \tau_i \qquad \forall i \qquad \text{(H9)}$$

However, testing this hypothesis is infeasible since it requires 2N + 1 parameters to be estimated but only 2N independent pieces of information are available in the standard system of equation regression setting. Faff *et al.* (2000) overcome this obstacle by applying the test to zero dividend firms, for which the individual tax credits are zero (i.e. $\tau_i = 0$). Thus, in this restricted setting the null hypothesis simplifies to:

$$H_0: \alpha_i = \beta_i \tau_m \qquad \forall i \qquad \text{(H9)}$$

Similar to the earlier sections, this model can also be tested in its zero-beta form. To this end, Faff *et al.* (2000) show that the counterpart hypothesis to H9, becomes:

$$H_0: \alpha_i = r_z(1 - \beta_i) + \beta_i \tau_m \qquad \forall i \qquad \text{(H10)}$$

Again we see in setting up these null hypotheses, alpha fully captures the asset pricing information (i.e. alpha is non-zero, taking a very special form). An example of this type of testing approach can be found in Faff *et al.* (2000). Faff *et al.* (2000) apply the test to an Australian sample using monthly data, over the period 1974 to 1995. The chosen test assets are beta, industry, size and dividend yield portfolios.

4.4 Intertemporal CAPM (ICAPM)

Merton (1973) assumes that investors face a multi-period world, rather than the single-period setting upon which CAPM is founded. In such a multi-period setting investors maximize expected utility of their life-time consumption. Hence, in addition to their diversification needs, they will now also be concerned about unfavorable shifts in the investment opportunity set over time. Merton shows that in equilibrium they will require compensation for systematic market risk (as in the standard CAPM), but now also compensation for the risk associated with uncertainty surrounding future investment opportunities. This latter concern gives rise to a hedging motive for investors when constructing their portfolios.

Formally, the ICAPM is expressed as:

$$E(r_i) = \beta_{im} E(r_m) + \beta_{ih} E(r_h) \qquad \text{(T11)}$$

where $E(r_h)$ is the expected excess return on the hedging portfolio. The empirical counterpart is given by:

$$r_{it} = \alpha_i + b_{im} r_{mt} + b_{ih} r_{ht} + \varepsilon_{it} \qquad \text{(E7)}[11]$$

The zero alpha test (as shown above, reflected in hypothesis H2) is one way to test the model. The alternative means of testing requires us to re-cast the setting in a way that will force the pricing information into the alpha terms. To achieve this, we re-express the ICAPM as follows (i.e. in raw returns):

$$E(R_i) = \gamma_0 + \beta_{im}\gamma_1 + \beta_{ih}\gamma_2 \qquad \text{(T12)}$$

where γ_0, γ_1 and γ_2 are three pricing parameters: γ_0 is the risk-free return if it exists, otherwise it becomes the expected ICAPM zero-beta return; γ_1 is the market price of (risk premium relating to) market beta; and γ_2 is the market price of (risk premium relating to) the hedging beta.

The empirical counterpart is given by:

$$R_{it} = \alpha_i + b_{im}R_{mt} + b_{ih}R_{ht} + \varepsilon_{it} \qquad \text{(E8)}$$

and similar to the CCAPM setup, both RHS variables in (E8) are assumed to be (sample) mean-adjusted. Thus, taking sample expectations through (E8) gives:

$$\bar{R}_i = \alpha_i \qquad \text{(E9)}$$

Thus, comparing the ICAPM reflected in (T12) with the empirical counterparts in (E8) and (E9), the theoretical restriction, and hence the null hypothesis for the ICAPM is given by:

$$H_0\colon \alpha_i = \gamma_0 + \beta_{im}\gamma_1 + \beta_{ih}\gamma_2 \qquad \forall\, i \qquad \text{(H11)}$$

Once more, note that this is a multivariate, non-linear cross-equation restriction – the package of alphas are designed to fully capture the relevant asset pricing information. If the data "comply" to the economic model (i.e. the ICAPM), tests will be unable to reject the theoretical restriction shown in H11.

An example of this type of testing approach can be found in Faff & Chan (1998). They apply the test to an Australian sample using monthly data, over the period 1975 to 1994. The chosen test assets are 24 industry portfolios.

4.5 Fama-French Three-factor Model

The Fama and French (1993) model is specified as:

$$E(R_i) - R_f = b_i\,[E(R_m) - R_f] + s_i\,E(SMB) + h_i\,E(HML) \qquad \text{(T13)}$$

where E(SMB) is the expected return on the mimicking portfolio for the size factor and E(HML) is the expected return on the mimicking portfolio for the book-to-market factor.

The empirical counterpart to this model is:

$$r_{it} = \alpha_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it} \qquad \text{(E10)}$$

Similar to the test derived in Section 2, the null hypothesis predicts joint zero mispricing across all I test assets i.e. H_0: $\alpha_i = 0$; i = 1, 2, ... I.

The Fama-French model can be re-expressed in terms of pricing parameters:

$$E(r_i) = b_i \lambda_m + s_i \lambda_{SMB} + h_i \lambda_{HML} \qquad \text{(T14)}$$

where λ_m, λ_{SMB}, λ_{HML} are the premia (or market prices) attaching to the associated Fama-French factors. Further, if we assume that the empirical RHS variables in equation (E10) are (sample) mean-adjusted, similar to the cases above for the CCAPM and ICAPM, the null hypothesis is now a test of the intercept terms conforming to the following non-linear cross-equation restriction:[12]

$$H_0: \alpha_i = b_i \lambda_m + s_i \lambda_{SMB} + h_i \lambda_{HML} \quad \forall i \qquad \text{(H12)}$$

Yet again the empirical design is setup such that the pricing information is collectively captured in the full set of alpha terms.

An example of this type of testing approach can be found in Faff (2003). He applies the test to US sample using monthly (daily) data, over the period 1979 to 1999 (1995 to 1999). The chosen test assets are 71 (77) industry portfolios, respectively, defined and constructed by DataStream.

4.6 Augmented Fama-French Models

The Fama-French model can be augmented by particular additional factors that researchers believe might be priced.[13] For example, Chan & Faff (2005) consider a liquidity factor, while Chan *et al.* (2011) consider a default risk factor. The nature of the tests take a very similar form to the previous sub-section. In terms of the "economic" asset pricing model, we have the default risk augmented model:

$$E(r_i) = b_i \lambda_m + s_i \lambda_{SMB} + h_i \lambda_{HML} + d_i \lambda_{DEF} \qquad \text{(T15)}$$

and the liquidity risk augmented model:

$$E(ri) = bi\,\lambda m + si\,\lambda SMB + hi\,\lambda HML + li\,\lambda IMV \qquad \text{(T16)}$$

where λ_{DEF} and λ_{IMV} are the premia (or market prices) attaching to the default and illiquidity factors, respectively.

The empirical counterparts to these models are:

$$r_{it} = \alpha_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + d_i DEF_t + \varepsilon_{it} \quad \text{(E11)}$$

$$r_{it} = \alpha_i + b_i(R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + l_i IMV_t + \varepsilon_{it} \quad \text{(E12)}$$

where DEF_t (IMV_t) is the return on the mimicking portfolio for the default risk (illiquidity) factor in period t. As before, we assume that the empirical RHS variables in equations (E11) and (E12) are (sample) mean-adjusted and the null hypotheses are now respective tests of the intercept terms conforming to the following non-linear cross-equation restrictions:

$$H_0: \alpha_i = b_i\lambda_m + s_i\lambda_{SMB} + h_i\lambda_{HML} + d_i\lambda_{DEF} \quad \forall i \qquad \text{(H13)}$$

$$H_0: \alpha_i = b_i\lambda_m + s_i\lambda_{SMB} + h_i\lambda_{HML} + l_i\lambda_{IMV} \quad \forall i \qquad \text{(H14)}$$

With regard to the default risk case, Chan *et al.* (2011) apply a test of H13 to an Australian sample using monthly data, over the period 1975 to 2004. The chosen test assets are 27 portfolios, based on independent tercile splits on size, book-to-market and a default likelihood indicator. With regard to the illiquidity case, Chan & Faff (2005) apply a test of H14 to an Australian sample using monthly data, over the period 1990 to 1998. The chosen test assets are 27 portfolios, based on independent splits on size, book-to-market and liquidity.

5. Concluding remarks

A fundamental pillar of modern finance theory is the risk-return relation, and traditionally alpha is taken to represent the degree of "mispricing" in asset returns. But, such an interpretation is not always appropriate – for certain empirical designs, a non-zero alpha embodies pricing information. In this paper, I have explained and illustrated the distinguishing circumstances between these two diametrically opposed cases.

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Appendix

From the main text we have:

$$E(R_i) = R_z(1 - \beta_i) + \beta_i E(R_m) \qquad \text{(T6)}$$

Begin by subtracting $R_f(1 - \beta_i)$ from both sides of (T6) to give:

$$E(R_i) - R_f(1 - \beta_i) = R_z(1 - \beta_i) + \beta_i E(R_m) - R_f(1 - \beta_i) \qquad \text{(T6.1)}$$

Then, regarding the LHS of equation (T6.1), expand the term:

$$-R_f(1 - \beta_i) = -R_f + R_f\beta_i \qquad \text{(T6.2)}$$

and substitute in equation (T6.1):

$$E(R_i) - R_f + R_f\beta_i = R_z(1 - \beta_i) + \beta_i E(R_m) - R_f(1 - \beta_i) \qquad \text{(T6.3)}$$

Now on the RHS collect the terms involving $(1 - \beta_i)$:

$$E(R_i) - R_f + R_f\beta_i - (R_z - R_f) * (1 - \beta_i) + \beta_i E(R_m) \qquad \text{(T6.4)}$$

Then utilising the definition of the excess return on the zero-beta asset, i.e. $r_z = R_z - R_f$; substitute in equation (T6.4):

$$E(R_i) - R_f + R_f\beta_i = r_z(1 - \beta_i) + \beta_i E(R_m) \qquad \text{(T6.5)}$$

Now subtract the term $R_f\beta_i$ from both sides of the equation and define the excess return variable on the LHS:

$$E(r_i) = r_z(1 - \beta_i) + \beta_i E(R_m) - R_f\beta_i \qquad \text{(T6.6)}$$

Finally, collecting terms on beta on the RHS, we get equation (T7) in the text:

$$E(r_i) = r_z(1 - \beta_i) + \beta_i E(r_m) \qquad \text{(T7)}$$

[1] For a very recent episode of this debate see the special issue of Abacus 2012 in which several authors debate whether the CAPM is a "failed revolutionary idea", in the context of a very early and famous empirical test of the model – namely, Black, Jensen and Scholes (1972). See, for example, Dempsey (2013), Benson & Faff (2013) and Smith & Walsh (2012).

[2] Harvey *et al.* (2014) document that in excess of 300 "factors" have been "discovered" in the literature, though they question whether (due to the collective effect of repeated sampling) appropriately high statistical cutoffs are being applied. Pukthuanthong & Roll (2014) propose a seven-stage protocol for identifying and measuring factors, that hopefully will assist asset pricing students and researchers decide which of the hundreds of factors can be safely ignored.

[3] For an amusing, but insightful, view of the new five-factor model from a 'real world" perspective, see DeMuth (2014).

[4] I distinguish three different types of equation or expression: (a) theoretical expressions coming from financial economic theory or established finance models (i.e. absent error terms); (b) empirical expressions (with random error terms) containing ex post variables and specifications designed to match off against theory; (c) expressions of hypotheses that come about as a result of comparing a given theoretical model to its empirical counterpart. To delineate the three types of expressions in the text, their numbering is prefixed with a "T, "E" and "H", respectively.

[5] Of course, we must acknowledge Roll's (1977) critique that argues since the true market portfolio is unobservable, testing the CAPM is extremely problematic.

[6] The term "test asset" is generic – it might refer to an individual stock or it might involve particular type of portfolio. Whatever the case, it is clear that in the asset pricing case, we are only interested in "passive" assets i.e. assets for which there is no managed element, since we would expect actively managed assets to contain considerable mispricing noise. For reasons of minimizing noise ("EIV" bias), the chosen set of test assets in the vast majority of asset pricing studies are portfolios.

[7] Indeed, the magnitude and sign of alpha can be interpreted as a measure of the extent to which the model fails to price a given test asset. For example, Faff (1992) documents an annualised average positive mispricing of around 25% (10%) p.a. for the smallest decile portfolio when testing the CAPM (APT).

[8] See the appendix for details.

[9] See Xiao *et al*. (2012).

[10] With the exception of the sign of the tax-related parameters, this formulation is identical to Brennan's (1970) classical tax based model.

[11] To apply (E7), a choice needs to be made regarding what empirical proxy(ies) will be used to capture the "hedge" portfolio(s). For example, Faff & Chan (1998) choose gold bullion returns.

[12] In this case, when expectations are taken through, all the RHS variables in (E10) have a value of zero and so drop out. This leaves just alpha.

[13] An important question in these situations is how to gain confidence that the selected factor is a good/legitimate choice. As mentioned earlier, Pukthuanthong & Roll (2014) propose a seven-stage protocol designed for this purpose.